A new determination of the inverse of a function

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Determining the inverse of a function is one of several applications to functions. In our high schools, the known method of computing the inverse of a function involves the following procedures;

- Multiplying both sides of the equation by the denominator of the function
- Expanding and thereafter, make x the subject
- Change the y variables in the result to x variables.

Following these procedures leads to computing the inverse of a function. Determining the inverse of a function is important tool in computing of the range of the function.

In this paper, we shall reveal a much simpler, intuitive, less time taken method of determining the inverse of a function by envisioning a right angled triangle about the function. This method of inverse of a function determination is what I call, "***the virtual triangle***" method.

Keywords: Functions, inverse of functions, right angled triangle, range of function.

Introduction:

Functions may appear with two x-terms or a single x-term. Below are examples of how may appear in questions;

Two x-term functions: $y = \frac{2x-7}{4+3x}$, $y = \frac{4x}{5+8x}$

Single x-term function: $y = \frac{7x-3}{5}$

In this method of determining the inverse of a function, the only tool required is just the mind, no need for a writing tool. In order to fully understand the virtual triangle method, the following formulations must be systematically adhered;

Formulations of the Virtual triangle

When presented with a question such as; determine the inverse of the function y = $\frac{2x-7}{4+3x}$

- First envision a right angled triangle about the function ALWAYS ensuring the x terms are connected. In the above example, the envisioned right triangle will be as such;

$$y = \frac{2x-7}{4+3x}$$

This shows the connection of the x-terms.

$$y = \frac{2x-7}{4+3x}$$

This shows the completion of right angle triangle.

The above right angle triangle must be pictured about the function ensuring the x-terms are always connected.

- Observe carefully the location or position of the x-terms. That is whether they are along the hypotenuse or vertical.

$$y = \frac{2x-7}{4+3x}$$

This envisioned triangle reveals the position of x-terms as along the hypotenuse

$$y = \frac{9x-7}{4x+3}$$

This envisioned triangle reveals the position of x-terms as along the vertical.

- If the x-terms are located along the hypotenuse, switch the position of coefficients or numbers only, along the vertical. If the x-terms are located along the vertical, switch the position of coefficients or numbers only along the hypotenuse.

$$y = \frac{2x-7}{4+3x}$$

From the envisioned right angled triangle above, the x-terms (2x and 3x) are located along the hypotenuse. From the formulation above, we switch the positions of the coefficients or numbers on the vertical. The numbers along the vertical are (2 and 4) as shown below;

$$y = \frac{2x - 7}{4 + 3x}$$

Numbers along the vertical (2 and 4).

The formulation said*; switch the positions of numbers along the vertical when x-terms are located along the hypotenuse*. Thus from the diagram above, we switch the positions of 2 and 4 to get;

$$y = \frac{2x - 7}{4 + 3x} \longrightarrow y = \frac{4x - 7}{2 + 3x}$$

- Negate the numbers or coefficients switched.

The switched function as seen above is $y = \frac{4x - 7}{2 + 3x}$ and the numbers we switched were (2 and 4), thus from the final formulation above, we negate the numbers we switched to conclude the virtual triangle method of computing the inverse of a function.

Negating the numbers or coefficients, we have $\boldsymbol{y = \frac{-4x - 7}{-2 + 3x}}$

This final result is the inverse of the function.

<u>Functions with three parameters</u>

The example shown above, involves a four parameter function. That is a function with two x-terms and two numbers. In this section of the paper, we shall discuss how to apply the formulations given above to compute the inverse of a function containing three parameters.

A three parameter function can be one of the following;

$y = \frac{4x}{5+8x} \longrightarrow$ Three parameter function with two x-terms and a number.

$y = \frac{7x-3}{5} \longrightarrow$ Three parameter function with two numbers and an x-term.

In order to apply the formulations described above to determine the inverse of a three parameter function, a fourth term must be introduced into the function in order to make it a four parameter function such as the one described above.

With a three parameter function with two x-terms, the fourth parameter to be added is zero (0) and for a three parameter with two numbers, the fourth parameter to be added is zero x (0x).

In a question such as; determine the inverse of the function, y $= \frac{7x-3}{5}$

<u>Solution</u>:

This is a three parameter function with two numbers and a single x-term. We have to introduce a fourth term which as described above must be 0x in order to effectively apply the formulations of the virtual triangle successfully.

Introducing the fourth term, 0x, we now have, $y = \frac{7x-3}{5+0x}$. We can now apply the formulations of the virtual triangle to determine the inverse of the function.

- Envision a right angled triangle about the function

$$y = \frac{7x-3}{5+0x}$$

From the envisioned right angled triangle above, the x terms (7x and 0x) are located along the hypotenuse, thus switch the positions of the numbers or coefficients along the vertical, which are (7 and 5).

$$y = \frac{7x-3}{5+0x} \qquad \longrightarrow \qquad y = \frac{5x-3}{7+0x}$$

Negate the numbers switched to conclude the process of computing the inverse of the function given above.

$$y = \frac{5x-3}{7+0x} \qquad \longrightarrow \qquad y = \frac{-5x-3}{-7+0x} \quad \text{or} \quad y = \frac{-5x-3}{-7}$$

The inverse of the function y $= \frac{7x-3}{5}$ is $\mathbf{y = \frac{-5x-3}{-7}}$ **.**

<u>Conclusion</u>:

The motive behind this new module is to reduce the time taken by students to solve questions involving inverse of functions or range of functions. The formulations described above may seem lengthy but just when the right angled triangle is envisioned, the rest is easy, as numbers are switched and negated.